San Diego Math League Middle School Division, Round 2a
January 26, 2008

1. If the reciprocal of $x - 7$ is $x + 7$, then $x$ is equal to
   (A) $\pm 7$ (B) $\pm 4\sqrt{3}$ (C) $\pm 5\sqrt{2}$ (D) $49 \pm 1$ (E) $\pm 50$

2. A cube has surface area 150 cm$^2$. Its volume in cm$^3$ is
   (A) $5\sqrt{3}$ (B) 5 (C) 125 (D) 216 (E) 25

3. Starting at 13 and counting up by 8, a student counts 13, 21, etc. A number that will be counted is
   (A) 34 (B) 48 (C) 53 (D) 63 (E) 71

4. A circular pizza is sliced into pieces using straight cuts. What is the maximum number of pieces that can be obtained with 5 cuts?
   (A) 10 (B) 15 (C) 16 (D) 17 (E) 18

5. The number 32 can be expressed as the sum of two prime numbers in two different ways. The sum of the smallest two of these four primes is
   (A) 2 (B) 4 (C) 8 (D) 16 (E) 32

6. If a fair coin is tossed three times, then what is the probability that exactly two tails appear?
   (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{3}{8}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

7. If $\frac{a}{x-b} = \frac{b}{x-a}$ and $a \neq b$, then $x$ is equal to
   (A) $b - a$ (B) $a - b$ (C) $a + b$ (D) 1 (E) $-a - b$

8. The surface of a rectangular $4 \times 3 \times 2$ block is painted red, and the block is then cut into cubes with side length 1. The number of cubes that have exactly one red face is
   (A) 0 (B) 4 (C) 8 (D) 12 (E) 24

9. Let $C_1$ and $C_2$ be two circles with the same center, such that circle $C_1$ is inside circle $C_2$. A line is tangent to circle $C_1$ and intersects circle $C_2$ at $A$ and $B$. If $AB = 2$, then the area between the two circles is
   (A) $\pi$ (B) 2 (C) 4 (D) $\frac{\pi}{2}$ (E) $4\pi$

10. Which of the following five numbers is smallest?

$$\frac{1}{0.125}, \quad \left(\frac{1}{4^{2/3}}\right)^3, \quad \frac{4^{3/2}}{8}, \quad (\sqrt[6]{4})^2, \quad \left(\frac{1}{2^{-1}}\right)^{-4/3}.$$

   (A) $\frac{1}{0.125}$ (B) $\left(\frac{1}{4^{2/3}}\right)^3$ (C) $\frac{4^{3/2}}{8}$ (D) $(\sqrt[6]{4})^2$ (E) $\left(\frac{1}{2^{-1}}\right)^{-4/3}$

11. In her latest game, Mary bowled 199 and this raised her average from 177 to 178. To raise her average to 179 with the next game, she must bowl

    (A) 179 (B) 180 (C) 199 (D) 200 (E) 201

12. A positive integer is called *ascending* if each digit in the number (except the first) is greater than the digit on its left. For example, 2478 is an ascending number. The number of ascending numbers between 4000 and 5000 is

    (A) 6 (B) 8 (C) 9 (D) 10 (E) 15

13. If $n$ is a positive integer, then an integer that is always divisible by 3 is

    (A) $(n+1)(n+4)$ (B) $n(n+2)(n+6)$
    (C) $n(n+2)(n+4)$ (D) $n(n+3)(n-3)$
    (E) $(n+2)(n+3)(n+5)$

14. A club with 100 members has a telephone call system. Certain members of the club, including the president of the club, maintain a list of up to three people. To make an announcement, the president calls each person on his list. In turn, every member who has a list and receives a call forwards the message to everyone on his or her list. If all 100 members can be contacted under this system, then the maximum number of members who do not have to have a list is

    (A) 33 (B) 34 (C) 66 (D) 67 (E) 75

15. If $x^2 + 4x + 6$ is a factor of $x^4 + rx^2 + s$, then $r + s$ is

    (A) 10 (B) 32 (C) 40 (D) 42 (E) 52

San Diego Math League Middle School Division, Round 2b
January 26, 2008

1. If $\frac{7a-5b}{b} = 7$, then find the value of $\frac{4a+6b}{2a}$.
2. How many digits are in the number $2^{14} \cdot 5^{11}$?
3. The numbers $\frac{w}{2}$, $w+2$, $2w$, $x$, $y$, $z$ form an arithmetic sequence. Find $x+y+z$.
4. Steve counted the number of digits among the page numbers of his history textbook, and found that there were a total of 726 digits. What is the highest page number?
5. Let $x$, $y$, and $z$ be integers such that $1 \leq x \leq 6$, $7 \leq y \leq 13$, and $14 \leq z \leq 21$. For these ranges of $x$, $y$, and $z$, how many solutions does the equation $x+y+z = 28$ have?
6. Let $XYZ$ be a triangle, and let $A$ be the midpoint of $YZ$, and let $B$ be the midpoint of $XY$. If $\angle XYZ = 90^\circ$, $XA = 9$, and $ZB = 7$, then find $XZ$.
7. The positive integer 7654321 is *not* divisible by 11. Find the greatest positive integer that can be formed by changing the order of the seven digits such that this new number is divisible by 11.
8. Quadrilateral $ABCD$ has right angles at $B$ and $D$. If $AB = AD = 20$ and $BC = CD = 15$, then find the radius of the circle inscribed in quadrilateral $ABCD$.